An analytical-numerical method for determining the mechanical response of a condenser microphone

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(Received 12 May 2011; revised 16 September 2011; accepted 16 September 2011)

The paper is based on determining the reaction pressure on the diaphragm of a condenser microphone by integrating numerically the frequency domain Stokes system describing the velocity and the pressure in the air domain beneath the diaphragm. Afterwards, the membrane displacement can be obtained analytically or numerically. The method is general and can be applied to any geometry of the backplate holes, slits, and backchamber. As examples, the method is applied to the Bruel & Kjaer (B&K) 4134 1/2-inch microphone determining the mechanical sensitivity and the mechano-thermal noise for a domain of frequencies and also the displacement field of the membrane for two specified frequencies. These elements compare well with the measured values published in the literature. Also a new design, completely micromachined (including the backvolume) of the B&K micro-electro-mechanical systems (MEM) 1/4-inch measurement microphone is proposed. It is shown that its mechanical performances are very similar to those of the B&K MEMS measurement microphone.
 [DOI: 10.1121/1.3652853]

PACS number(s): 43.38.Kb, 43.38.Bs [AJZ] Pages: 3698–3705

I. INTRODUCTION

Predicting the response of a condenser microphone due to an incoming sound wave in terms of its geometrical and material properties is an almost century old problem. The difficulties in its approaches are related especially with the coupling of the vibration of the membrane with the oscillations of the underlying air layer. While the general motion of a membrane supported on a rigid circular frame at its periphery, driven by a given surface force is known [see Chapter 9 in the book by Rayleigh (Ref. 1, and the references therein)] the fluid motion in the air domain (including the gap, holes, slit and backvolume) is more difficult to describe analytically.

The first general approaches of the condenser microphone analysis were based on a simplified model using some lumped elements by Wente[2] in 1917 and Crandall[3] in 1918 using simplifying assumptions about the pressure and particle velocity in the air domain, as have been considered by Warren *et al*.[4] in their paper published in 1973.

A second approach was based on using the Navier-Stokes (N-S) system for describing the motion of the viscous fluid (air) coupled with the vibrating membrane. Thus, Robey[5] in 1954 tried to solve the N-S system in a cylindrical domain (the underlying air) but used the unphysical boundary condition of vanishing pressure on the external circular cylindrical surface. Petritskaya[6,7] in 1966–1968, improved Robey's solution by considering the appropriate boundary condition of vanishing fluid velocity on the external circular surface. Also, she accounted for the presence of openings in the backplate. Zuckerwar[8] in 1978 made two key assumptions which enabled him to simplify the analysis and obtain a closed form solution. He assumed that the membrane displacement is axi-symmetrical and that the reaction pressure is relatively insensitive to the shape of the diaphragm. The excellent agreement between the theory and experiment in the case of the Bruel & Kjaer (B&K) 1-in. and 1/2-in microphones shows that Zuckerwar's theory describes accurately the microphone behavior. A complete analysis for the B&K type 4146 1-in. microphone is also included in Ref. 9. Tan and Miao[10] in 2006 reviewed the theory of condenser microphones and proposed a new analytical modeling method for the B&K MEMS condenser microphone. The theoretical results obtained by this method were found to be in very good agreement with the experimental results reported in the case of a B&K MEMS microphone in Ref. 12. Very recently, Lavergne *et al*.[13] developed a theory for electrostatic acoustical transducers used in environments and/or frequency ranges which are significantly different from those for which they have been designed according to the previous described work. By using a lubrication-type approximation they obtained a Helmholtz-type equation for the pressure in the gap and used a similar equation for the pressure in the backchamber. Both equations contain delta functions as forcing terms, meant to describe the effect of the holes in the backplate and slit around the backplate.

In this paper, we propose a new approach based on a numerical solution of the Navier-Stokes system for the air domain $\mathcal{D}_{air}$ composed of the gap, the holes and slits in the backplate, the slit around the electrode and the backchamber. Thus, the nonlocal coupling of the pressure in the gap with the backchamber pressure through the holes in the backplate is taken into consideration in a natural way without any need to use the function defined by Petritskaya in Ref. 6.

The geometry used more often for building microphones is that of an axi-symmetrical backplate with regular azimuthally distributed circular holes. However, we can mention several cases approaching more general geometries. Thus, Skvor in

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Refs. 14 and 15 reported a theoretical and experimental study of a microphone having a nonplanar backplate. Also, Ref. 16 gives a simplified model of a microphone with square membrane and a nonplanar backplate.

The method presented in this work can be applied to general geometries of the backplate, holes, slits and backchamber. In the examples in this paper we consider firstly the case of a 1/2-inch B&K microphone with an axisymmetrical backplate having regular azimuthally distributed circular holes. Another example, a MEMS microphone, involves an octagonal backplate and square holes having still a sectorial periodicity. In the general case, the numerical procedure has to be applied to the whole diaphragm and not to only to a slice of it. The only difference is that the number of degrees of freedom will be much larger.

II. THE MEMBRANE EQUATION. THE ZERO ORDER SOLUTION

A. The membrane equation

Consider the cylindrical system of coordinates (r, θ, z) having its origin at the center O of the rigid circular frame bounding the membrane, and Oz-axis perpendicular to the equilibrium position of the membrane and directed outwardly. The radius of the frame is denoted by a and the origin of the azimuthal coordinate θ is set at the center of a hole in the backplate.

In the case of harmonic time variation $e^{i\omega t}$ of the incoming wave, the motion of the diaphragm is described by the membrane equation which in cylindrical coordinates can be written as

$$\nabla^2\eta(r,\theta) + k^2\eta(r,\theta) = -\frac{p_i}{T} + \frac{p(r,\theta,0)}{T}, \quad 0 < r < a \tag{1}$$

Here, $\eta\ (r, \theta)$ is the vertical membrane displacement and p_i the incident sound pressure assumed uniform (constant) over the membrane surface. Also, $p\ (r, \theta, 0)$ denotes the reaction pressure at the membrane surface, and k is the wave number of sound in the membrane,

$$k = \frac{\omega}{c}, \quad c = \sqrt{\frac{T}{\sigma_M}} \tag{2}$$

where σ_M is the membrane mass surface density, T the membrane tension and c denotes the sound speed in the membrane. The reaction pressure $p(r, \theta, 0)$, loading the diaphragm, is the pressure due to the underlying air layer squeezed between the membrane and backplate. Equation (1) has to be completed by the boundary condition

$$\eta(a,\theta) = 0 \tag{3}$$

stating that the membrane is supported by the rigid circular frame at its periphery.

B. The zero order solution for the membrane displacement

The zero order solution for the diaphragm motion is obtained by neglecting in Eq. (1) the reaction pressure. The solution of the problem of a circular membrane driven by the constant pressure p_i due to the sound field can be found in the book by Blackstock[17] (see p. 403). In our case it can be written as

$$\eta^{(0)} = \frac{p_i}{k^2 T}\left[\frac{J_0(kr)}{J_0(ka)} - 1\right] \tag{4}$$

The very same solution enters also as the first term in the solution given by Zuckerwar[8] in Eq. (17) and by Lavergne *et al.*[13] in Eq. (11a). We note that this solution is not influenced at all by the presence of the backplate and, consequently, does not depend on the azimuthal coordinate θ.

III. THE NAVIER-STOKES SYSTEM FOR THE AIR DOMAIN $\mathcal{D}_{air}$ THE FIRST ORDER APPROXIMATION OF THE REACTION PRESSURE

A. The linearized compressible Navier-Stokes system

The motion of the air in the domain underneath the membrane $\mathcal{D}_{air}$ composed of gap, holes and slits set in the backplate, peripheral slit surrounding it and backchamber is described by the Stokes approximation of the N-S compressible system written in the case of time harmonic dependence as

$$i\omega\rho_0\mathbf{v} = -\nabla p + \mu\left(\nabla^2\mathbf{v} + \frac{1}{3}\nabla\nabla\cdot\mathbf{v}\right) \tag{5}$$

$$\rho_0\nabla\cdot\mathbf{v} = -i\omega\rho \tag{6}$$

and the equation of state

$$p = \rho c_T^2 \tag{7}$$

Here, $\mathbf{v}$ is the air particle velocity, ρ_0 is the static air density, ρ is the time-varying air density, and c_T is the isothermal speed of sound in air. According to Crandall[3] the use of the isothermal sound speed c_T is appropriate for the gap region.

The boundary conditions associated with the system, Eqs. (5)–(7), are

$$\upsilon_r(r,\theta,0) = \upsilon_\theta(r,\theta,0) = 0, \quad \upsilon_z(r,\theta,0) = i\omega\eta(r,\theta) \tag{8}$$

along the membrane. Beside these, the velocity is zero,

$$\mathbf{v}|_S = 0 \tag{9}$$

on the all immobile solid surface boundaries S of the fluid domain (no slip condition).

The equations describing the motion of the diaphragm, Eq. (1), and fluid flow, Eqs. (5)–(7), are coupled. Thus, the fluid motion enters in the membrane equation by the reaction pressure $p(r, \theta, 0)$ while the diaphragm displacement influences the fluid flow by means of the boundary condition, Eq. (8). To solve the problem we consider some iterations.

B. The first order approximation for the reaction pressure. Computational methodology

For obtaining the first approximation for the reaction pressure we use in the third boundary condition, Eq. (8), the zeroth order approximation for the diaphragm displacement η^0 given by formula, Eq. (4). Thus, we write this boundary condition as

$$\upsilon_z^0(r,\theta,0) = \frac{i\omega}{k^2T}pi\left[\frac{J_0(kr)}{J_0(ka)} - 1\right] \quad (10)$$

We consider the case where the stationary backplate, containing the holes, can be divided into a number of N_0 identical circular sectors which can be obtained by rotation of the basic sector by an angle equal to a multiple of $2\pi/N_0$. As the forcing sound pressure is assumed constant over the diaphragm, the boundary conditions for each circular sector are the same. Consequently, the pressure will be a periodic function with respect to the azimuthal angle θ of period $2\pi/N_0$. Thus, the microphones B&K 4134 (1/2 in.) (top view in Fig. 1) and B&K 4146 (1 in.) studied by Zuckerwar in Refs. 8, 9, and 13 and the microphone type WS2 presented in Ref. 11 have $N_0 = 6$ while the B&K MEMS (1/4 in.) measurement microphone described in the paper by Scheeper *et al.*[12] has $N_0 = 4$. Similar cases were considered by Petritskaya[6] and Tan and Miao[10] Therefore, it is sufficient to determine the pressure in a basic sector. The modeled domain can be further reduced by accounting for the symmetry of the basic sector.

Finally, we need to build a finite element method (FEM) model only for the air domain $\mathcal{D}^0_{air}$ lying under the upper half of the basic domain of diaphragm $D^0 = \{(r,\ \theta,\ 0)|0 < r < a;\ 0 < \theta \le \pi/N_0;\}$. Inside the domain $\mathcal{D}^0_{air}$, Eqs. (5)–(7) prove true while on the surface $\partial\mathcal{D}^0_{air}$ the boundary conditions will be: symmetry conditions on the planes underneath the straight line segments on the boundary of the domain D^0, zero velocity on any part of solid boundaries of holes, slits and back-chamber and $\upsilon_r(r,\ \theta,\ 0) = \upsilon_\theta\ (r,\ \theta,\ 0) = 0$ and $\upsilon_z(r,\ \theta,\ 0)$ equal to the value given by relationship, Eq. (10), on the boundary of the air domain corresponding to the membrane.

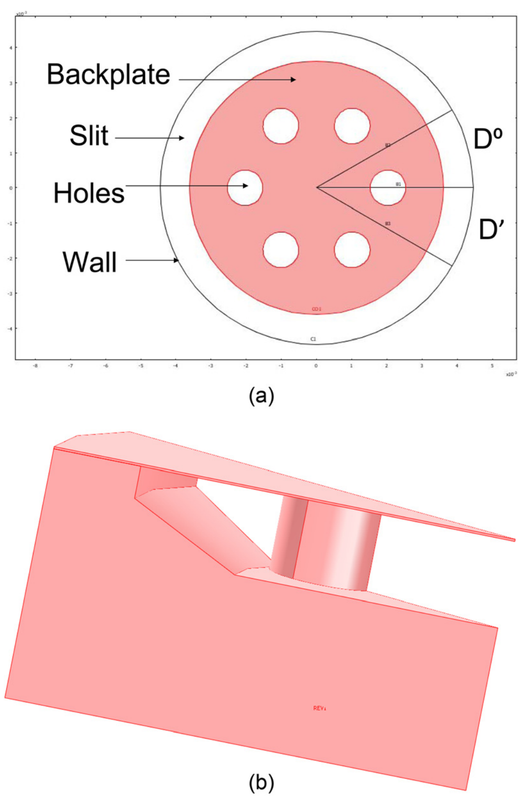

FIG. 1. (Color online) (a) Top view of the B&K 4134 microphone after removing the diaphragm. D^0 is the basic domain and D' its symmetrical domain. (b) The air domain D^0_{air} corresponding to the diaphragm domain D^0.

In this section, we will develop a numerical model based on the finite element method (FEM) for determining the reaction pressure on the diaphragm. The frequency domain linearized N-S, Eqs. (5)–(7) were implemented in the commercial finite element method code, COMSOL Multiphysics v.3.5. Thus, we used in the MEMS module in the program *Non-isothermal Stokes Flow*. The steady-state analysis corresponds to the system, Eqs. (5), (6), and (7), where $\omega = 0$. In order to take into consideration the term, $i\omega\rho_0\mathbf{v}$ in Eq. (5) and the term $i\omega\rho$ in Eq. (6), we introduce the corresponding terms in the weak form of the Stoke's system of equations implemented in COMSOL. A mesh of unstructured tetrahedral elements using $Lagrange - P_2P_1$ interpolation polynomials with a regular refinement at the gap was used. The resulting system of equations was solved with the parallel direct linear solver PARDISO. All the numerical work has been performed on a workstation having 96 Gbyte RAM and 24 cores. The typical time consumption for one frequency and a system of 123 295 degree of freedom (DOF) was around 50 s.

The frequency limitation of this approach is given by the condition that the number of DOF to be larger than $N_{DOF} = 1728\times$model volume measured in wavelength cubed. This condition is a strengthening of the requirement to have at least two DOF (practically 10–12) per wavelength in the direction of propagation (see Ref. 19). In our case the solution obtained is valid up to 250 kHz.

As a result of computation we obtain p^{calc} and hence the reaction pressure on the membrane basic domain D^0 can be written as

$$p^{(1)} = p_i p^{calc}.$$

IV. THE FIRST ORDER SOLUTION FOR THE DIAPHRAGM DISPLACEMENT

Once the first order approximation $p^{(1)}\ (r,\ \theta)$ of the reaction pressure of the diaphragm is determined, the first order displacement results by solving the equation

$$\nabla^2\eta^{(1)}(r,\theta) + k^2\eta^{(1)}(r,\theta) = -\frac{p_i}{T} + \frac{p_i}{T}p^{calc}(r,\theta,0) \quad (11)$$

with the perimeter condition

$$\eta^{(1)}(a,\theta) = 0. \quad (12)$$

In this section, we present two methods for obtaining the diaphragm displacement: an analytical solution based on the work of Rayleigh[1] and a numerical solution using the FEM method.

A. The analytical solution

As an analytical solution to this problem we will apply the method given by Morse and Ingard[18] (Section 9.3; pp. 543–544). Thus, in the case of a circular membrane, the even part, with respect to θ, of the diaphragm displacement can be written as

$$\eta^{(1)}(r,\theta) = \frac{p_i}{k_2 T}\left[\frac{J_0(kr)}{J_0(ka)} - 1\right] + \frac{pi}{\pi a^2 T} \times \sum_{m,n} \frac{2-\delta_{m,0}}{\left(k^2 - k_{mn}^2\right)} \frac{\eta_{mn}(r,\theta)}{\left[J'_m(\alpha_{mn})\right]^2} \times \int_0^{2\pi}\int_0^a p^{calc}(r',\theta')\eta_{m,n}(r',\theta')r'dr'd\theta', \quad (13)$$

where

$$\eta_{m,n}(r,\theta) = J_m\left(\frac{\alpha_{mn}}{a} r\right)\cos(m\theta), \quad (14)$$

and the constants k_{mn} are given by relationships

$$J_m(\alpha_{mn}) = 0, \quad k_{mn} = \alpha_{mn}/a \quad (15)$$

by means of the zeros of the Bessel function $J_m(r)$.

The average displacement of the membrane enters into several parameters. To determine it, we write

$$\left\langle \eta^{(1)}(r,\theta)\right\rangle \equiv \frac{1}{\pi a^2}\int_0^{2\pi}\int_0^a \eta^{(1)}(r,\theta) r dr d\theta = \frac{p_i}{k^2 T}\frac{J_2(ak)}{J_0(ak)} + \frac{2p_i}{\pi a^2 T}\sum_{n=0}^{\infty}\frac{1}{\alpha_{0n}J_1(\alpha_{0n})k^2 - k_{0n}^2} \times \int_0^a\int_0^{2\pi} p^{calc}(r,\theta)J_0(k_{0n}r) r dr d\theta \quad (16)$$

Remark 1. Equation (16) was proven for the case of diaphragms having a symmetry axis (Ox). As the integral of "sine" terms cancels out it is also valid in the case of microphones having circular diaphragms and arbitrarily shaped backplates.

In the case of diaphragms having a symmetrical pattern with N_0 circular sectors the integral can be written as

$$\int_0^{2\pi}\int_0^a p^{calc}(r,\theta)J_0(k_{0n}r) r dr d\theta = 2N_0 \int_{D^0}\int p^{calc}(r,\theta)J_0(k_{0n}r) r dr d\theta \quad (17)$$

B. A numerical solution

Once the reaction pressure on the membrane is known, the first order displacement of the diaphragm $\eta^1(r,\theta)$ can also be obtained by numerical integration of the Helmholtz Eq. (11). This can be performed by using the Multiphysics module of the COMSOL finite element package by coupling it with the program developed for determining the pressure on the diaphragm. For this we considered a nonstructured mesh of the 2D domain D^0 containing a total of 12 480 triangles and have used a Lagrange-quadratic interpolation. The resulting system was solved again by using the PARDISO direct solver. Being a 2D problem the time required for this calculation is significantly smaller than that necessary for determining the reaction pressure. After determination of the first order displacement its average value over the diaphragm is

$$\left\langle \eta^1(r,\theta)\right\rangle = p_i\frac{2N_0}{\pi a^2}\int\int_{D^0}\eta^1(r,\theta)dS \equiv \frac{p_i}{\pi a^2}J(\omega), \quad (18)$$

where by $J(\omega)$ has been denoted the integral of the first order displacement over the membrane surface:

$$J(\omega) = 2N_0\int\int_{D^0}\eta^{(1)}(r,\theta,\omega)dS. \quad (19)$$

V. ACOUSTICAL LUMPED ELEMENTS

A microphone can be viewed as a complex acoustical electromechanical system having tightly coupled acoustical, electrical and mechanical elements. A very much used modeling method for determining the performance of a microphone is the lumped-element approach in which simple analytical (or numerically determined) expressions for mass, compliance and damping have their equivalent corresponding electrical counterparts in inductance, capacitance and resistance, respectively. A standard representation for a capacitive sensor including the radiation impedance, the mechanical damping, stiffness and mass, and electrical characteristics as capacitance (including the parasitic one) and electrical resistance can be found in the book by Kinsler *et al.*[20] A simplified acoustical lumped-element equivalent circuit of the microphone is shown in Fig. 2. It includes the membrane mass M_M and *compliance* C_M and also the resistance R_A and the compliance C_A of the air in gap, holes and backchamber.

Finally, the mechanical performance of a condenser microphone consists of two parts: the mechanical sensitivity and the thermal noise.

1. Mechanical sensitivity

The membrane parameters can be written directly as

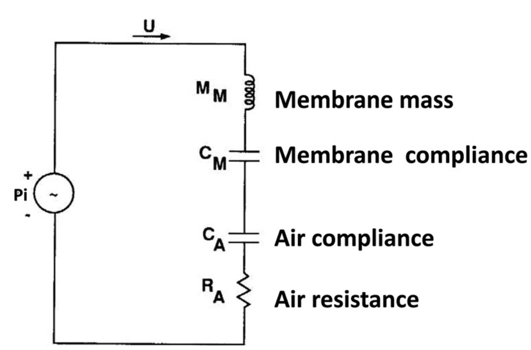

FIG. 2. The simplified acoustical lumped-element equivalent circuit of the microphone.

$$M_M = \frac{4\sigma_M}{3\pi a^2} \quad \text{the diaphragm mass,} \tag{20}$$

and

$$C_M = \frac{(\pi a^2)^2}{8\pi T} \quad \text{the diaphragm compliance.} \tag{21}$$

At frequencies that are below the first resonant frequency of the diaphragm the equation of the equivalent circuit of the microphone can be written as

$$(i\omega)^2 M_M + i\omega R_A + \left(\frac{1}{C_A} + \frac{1}{C_M}\right) = \frac{1}{\pi a^2 M_m(\omega)}. \tag{22}$$

The source term in the right hand side of Eq. (22) contains the *mechanical sensitvity* defined as

$$M_m(\omega) = \frac{\langle \eta^{(1)}(r,\theta)\rangle}{p_i} \equiv \frac{J(\omega)}{\pi a^2}. \tag{23}$$

Once mechanical sensitivity known, the real part of Eq. (22) determine the air compliance as

$$C_A(\omega) = \left[\text{real}\left(\frac{1}{J(\omega)}\right) + \frac{4\sigma_M \omega^2}{3\pi a^2} - \frac{8\pi T}{(\pi a^2)^2}\right]^{-1}. \tag{24}$$

Further on, the imaginary part of Eq. (22) yields the air resistance

$$R_A(\omega) = \frac{1}{\omega}\,\text{imag}\left(\frac{1}{J(\omega)}\right). \tag{25}$$

Finally, the *quality factor Q is* given by formula

$$Q = \frac{1}{R_A}\sqrt{M_M\left(\frac{1}{C_M} + \frac{1}{C_A}\right)}. \tag{26}$$

A. Mechanical-thermal noise

The random movement of the molecules in a gas at a certain temperature and surrounding a mechanical structure leads to random fluctuations in the energy transfer between structure and damping gas, which is generally referred to as mechanical-thermal noise. According to Gabrielson,[21] any mechanical system in thermal equilibrium can be analyzed for mechanical-thermal noise by adding a force generator alongside each damper. Because a thermal equilibrium between the mechanical device and the surroundings is assumed, the energy lost towards the environment through the dissipative friction (damping coefficient) must equal on average the energy gained through the noise force. In the case of a microphone the Nyquist theorem[21] relates the spectral density of the *fluctuating force* to the air resistance R_A

$$F_f = \sqrt{4k_\beta T^o R_A}[N/\sqrt{\text{Hz}}], \tag{27}$$

Or the *pressure spectral density of the mechanical-thermal noise* to the acoustic resistance $R_{acs} = R_A/S^2$ (where S is the area of the active force):

$$P_f = \sqrt{4k_\beta T^o R_{acs}}[N/\sqrt{\text{Hz}}], \tag{28}$$

where k_β is the Boltzmann constant (1.38×10^{-23} J/K) and T^o is the absolute temperature (K). The background noise mean square pressure (A-weighted) of the mechanical noise can be computed as

$$N_f^2 = \int_{f_1}^{f_2} 4k_\beta T^o R_A A^2(f)df, \tag{29}$$

where $A(f)$denotes the function of the A-weighted filter, and $f_1 = 10\,\text{Hz}, f_2 = 20\,\text{kHz}$.

VI. CALCULATION OF MECHANICAL PERFORMANCE OF SOME MICROPHONES

In this section, we shall compute the mechanical performance of the B&K 4134 and also of a new design for a MEMS microphone.

A. The B&K 4134 (1/2 in.) microphone

The arrangement of acoustical slit and holes in the backplate of the B&K microphone type 4134 is shown in Fig. 1(a). Figure 1(b) shows the corresponding air domain D^0_{air} used for determining the reaction pressure. The mechanical parameters of the membrane, the values of geometrical parameters of the microphone and the thermo-acoustical parameters of the air are those given by Zuckerwar.[8] The membrane has a radius $a = 4.45$[mm], the thickness 5[μm] the surface density $\sigma_m = 0.0445$[kg/m^{-2}] and the tension $T = 3162$[N/m]. Also, the radius of the circular backplate is 3.61[mm], the radius of the six holes equals 0.51[mm], the distance of the centers of the holes to the membrane center equals 2.03[mm]. The length (depth) of a hole is 0.84[mm] while the length of the peripheral slit is 0.3[mm]. Finally, the gap (the distance between average position of membrane and backplate) equals 20.77[μm] and the backchamber volume is 1.264×10^{-7}[m^3]. In all the calculations the density of the air is considered to be $\rho_0 = 1.2$[kg/m^3] its viscosity μ=1.89×10^{-5}[kg/m/s^{-3}] and the isothermal sound of speed in air $c_T = 290$[m/s].

Figure 3 compares the sensitivity obtained by using the reaction pressure obtained numerically and the two methods for solving the membrane equation: analytical method given in Section A (continuous line) and the numerical method of Section B (line with circles) with the measured values taken from Ref. 8 and shown in the two figures as empty circles. Figure 3(a) shows the variation of amplitude sensitivity with frequency of the incoming wave and Fig. 3(b) plots the variation of phase response with the same frequency. In both cases the analytical and numerically obtained results are very close. We note also that the Mechanical Sensitivity determined numerically is overdamped as compared with the measured values.

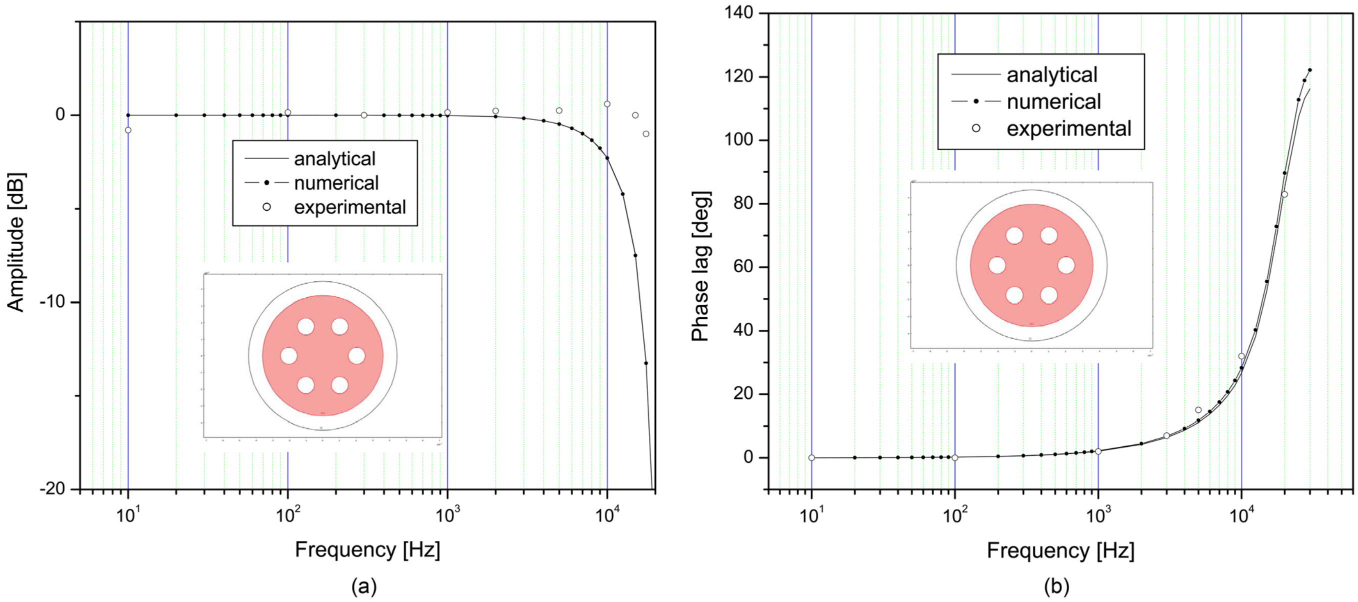

FIG. 3. (Color online) (a) Amplitude (dB) of the mechanical sensitivity of the B&K 4134 microphone, as a function of frequency, calculated analytically (continuous line) and numerically (line and circles) and the measured values (scattered empty circles). (b) The phase of the mechanical sensitivity of the microphone B&K 4134, as a function of frequency, calculated analytically (continuous line) and numerically (line and circles) and measured values (scattered empty circles).

Finally, by using the given formulas, we provide evaluation of the thermal noise. Thus, Eq. (25) yields the value $R_{acs} = 1.2 \times 10^8$ [Ns/m^5] for the mechanical resistance and hence the value $1.39 \times 10^{-6}[P/\sqrt{\text{Hz}}]$ for the pressure spectral density of the mechanical-thermal noise. This gives a mechanical thermal noise of 17.9[dB (A)]. This compares well with the values 19.7[dB (A)] corresponding to Zuckerwar's paper,[8] 18.9[dB (A)] given by Tarnow in Ref. 22, 18.3[dB (A)] found by Tan and Miao in Ref. 23, and the value 18.0[dB (A)] given in Ref. 24.

B. A new design of a MEMS microphone

The 1/4-inch, MEMS measurement microphone, of condenser type, developed by B&K[12] in 2003, has a very low level of mechanical-thermal noise comparable to the noise level corresponding to a 1/2-inch microphone discussed in the previous subsection. This low level of mechanical-thermal noise is ensured by a 20 [μm] gap between the diaphragm and the square backplate (which measures 2.8 $\times$ 2.8[mm^2]) the slit arround the backplate and the four acoustic square holes perforated in the backplate. Also, the high sensitivity of the microphone is achieved by using a quite large diaphragm for a MEMS microphone of $\varnothing = 3.9$[mm]. (The diaphragm radius is also limited by the design constraint of a 1/4-inch microphone housing). The micromachined diaphragm has the thickness 0.5[μm], the mass surface density $\sigma_m = 0.0015$ [kg/m^2] and tension $T = 170$[N/m]. Each square hole has the side length 80[μm], the distance of the centers of the holes to the membrane center equals 0.55[mm], and the depth (the backplate thickness) equals to 150[μm]. The microphone silicon chip contains the diaphragm and backplate and it is mounted onto a ring of silicone glue. For packaging and testing purposes, the

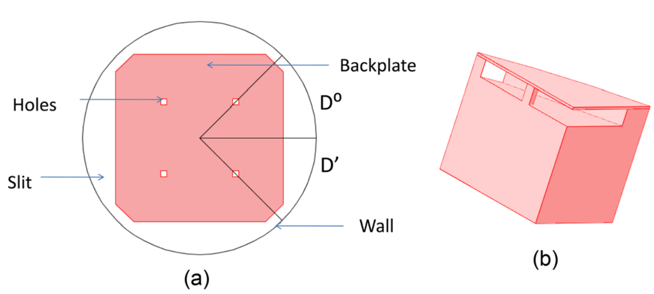

FIG. 4. (Color online) (a) Top view of the proposed MEMS design after removing the diaphragm. D^0 is the basic domain and D' its symmetrical domain. (b) The air domain D^0_{air} corresponding to the diaphragm domain D^0 for proposed MEMS design.

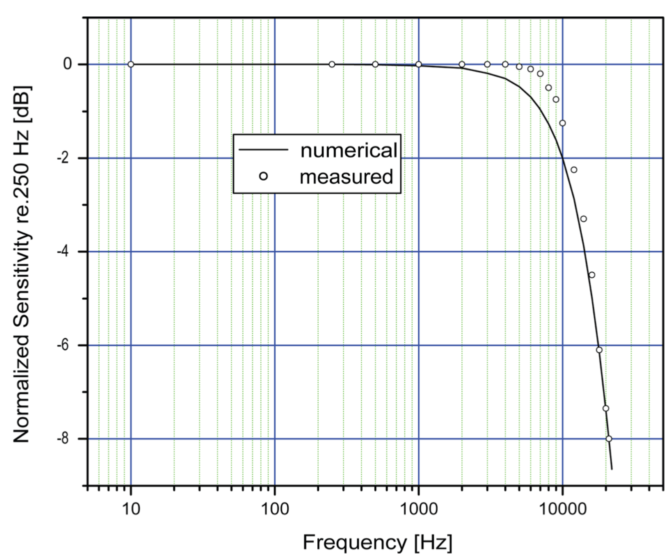

FIG. 5. (Color online) Amplitude (dB) of the mechanical sensitivity of the proposed microphone, as a function of frequency, calculated numerically (continuous line) and the measured values (scattered empty circles).

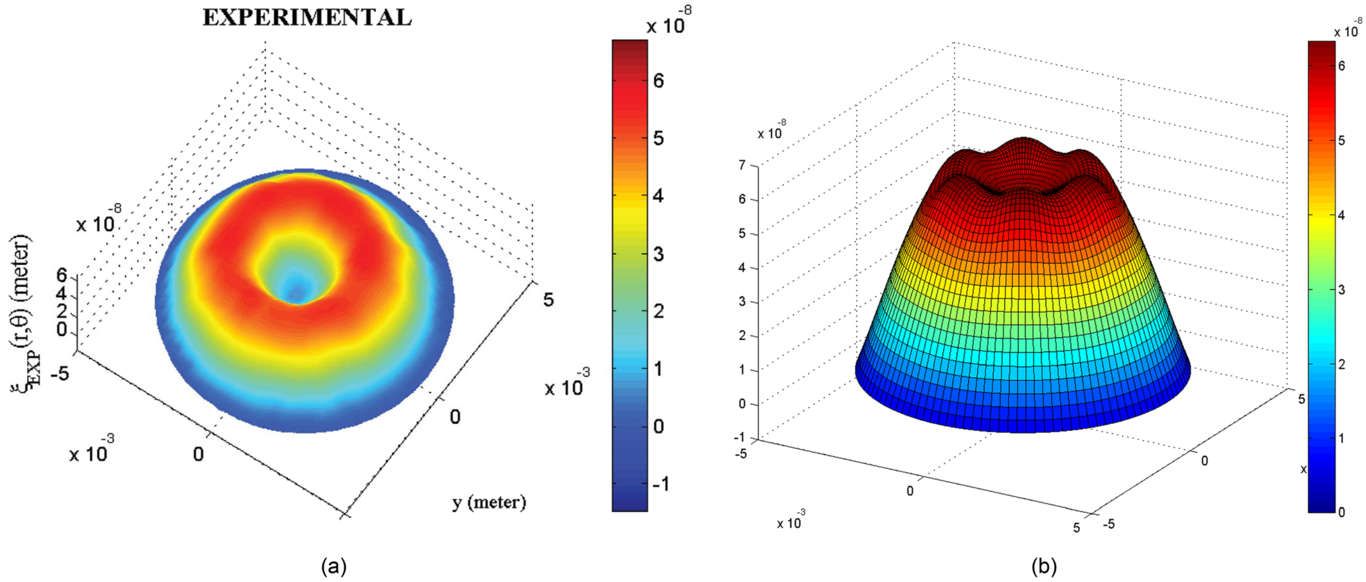

FIG. 6. (Color online) (a) Measured displacement field of the membrane of B&K 4134 microphone at 40 kHz reproduced from the paper by Lavergne *et al.*[13]. (b) The calculated displacement field of the membrane of B&K 4134 microphone at 40 kHz.

microphone is housed in a 1/4-inch microphone titanium body. This gives a backchamber volume of 76[mm^3] which corresponds to a height of 6.4 [mm].

For determining the mechanical performance of the microphone, by using the method developed by Zuckerwar, Miao *et al.*,[10] considered a model with a circular backplate and round holes. The analysis in paper[10] shows also that by modifying the gap to 10[μm] it is possible to build with the same technique new MEMS microphones for nonmeasurement applications.

In this section, we investigate the mechanical properties of a microphone completely micromachined, the silicon chip including also the backchamber. The diaphragm has the radius $a = 1.95$[mm], the octagonal backplate (see Fig. 1 in Ref. 12 and Fig. l(b) in Ref. 10) has the large sides length equal to 2.2[mm] and the distance between two opposite large sides of 2.8[mm] [Fig. 4(a)]. It contains four square acoustical holes which measure $80 \times 80[\mu m^2]$. The deep of the backchamber equals 0.8[mm]. Due to periodicity and symmetry the bounary-value problems have to be solved for the domain D^0 in Fig. 4(a) (for the diaphragm displacement) and the 3D domain shown in Fig. 4(b) for the pressure under the diaphragm. For both boundary value problems numerical methods have been applied.

The normalized sensitivity obtained by using the analyzed MEMS condenser microphone is plotted in Fig. 5, versus frequency of the incoming wave, as a continuous line. In the same figure the scattered empty circles correspond to the measured response of the MEMS measurement microphone given in Fig. 9 in Ref. 12. The good agreement of the

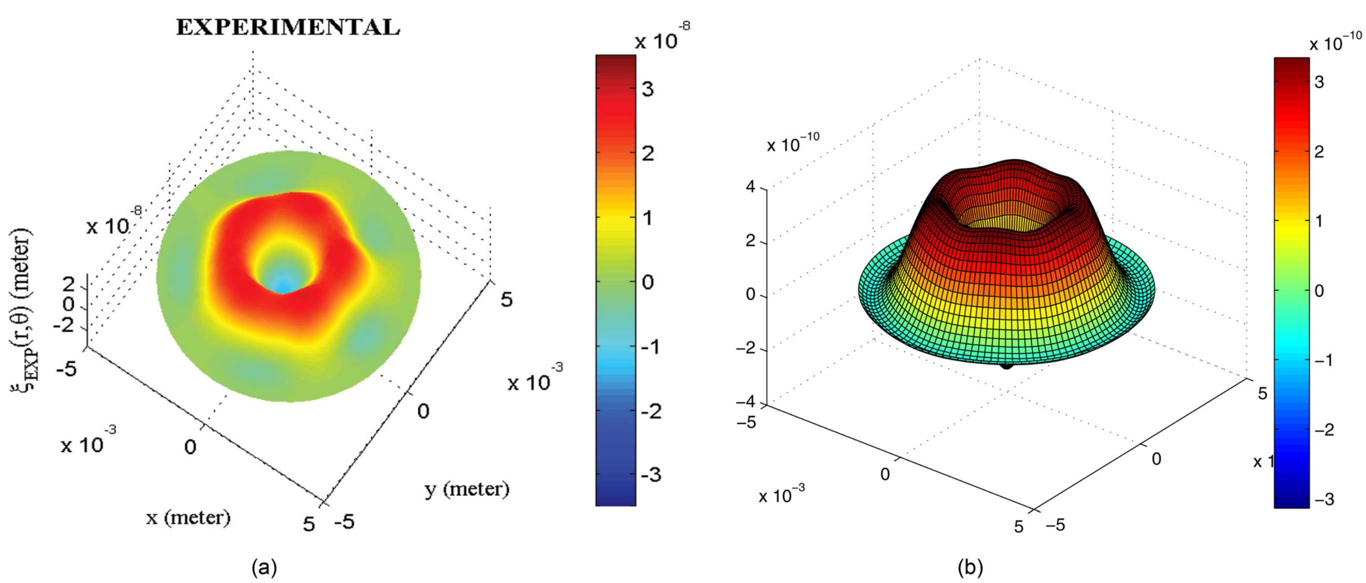

FIG. 7. (Color online) (a) Measured displacement field of the membrane of B&K 4134 microphone at 70 kHz reproduced from the paper by Lavergne *et al.*[13] (b) The calculated displacement field of the membrane of B&K 4134 microphone at 70 kHz.

calculated and measured data shows that the influence of the very deep backchamber in the MEMS measurement microphone it is not so important. Presumably, the shorter backchamber is responsible for the slight overdamping of the proposed MEMS microphone.

Now, by using the given formulas, Eqs. (25), (28), and (29), we can give evaluation of the thermal noise. Thus, we obtain $R_{acs} = 2.97 \times 10^8$[Ns/m^5] for the mechanical resistance. This gives a mechanical thermal noise of 21.8[dB(A)] which compares well with the values $20.8 - 21.7$[dB(A)] given in Table IV in Ref. 12 for the Thermal noise for B&K MEMS measurement microphones.

C. The displacement of membrane

The paper by Lavergne *et al.*[13] contains the displacement field of the B&K 1/2 in. microphone obtained experimentally using a scanning laser vibrometer. Figure 6(a) shows the displacement field of the membrane of the B&K 4134 microphone at 40 kHz, and the same field measured at 70 kHz is shown in Fig. 7(a). Both figures are taken from the cited paper. The displacement field of the membrane of the same microphone and for the same frequencies resulted by using the numerically obtained reaction pressure coupled with the numerical integration of the membrane equation are shown in Fig. 6(b) (for 40 kHz) and Fig. 7(b) for 70 kHz. In both cases the calculated values look very similar to the measured displacements.

VII. CONCLUSIONS

The main result of this work is the determination of the reaction pressure on the diaphragm due the air domain $\mathcal{D}_{air}$ underneath it by using a numerical approach based on the finite element method. Afterwards, the membrane displacement can be obtained analytically or numerically. In both cases the results are in very close agreement. The method is applied to determine the sensitivity and the mechano-thermal noise of the classical B&K 4134 1/2-inch microphone. Also, in the last section the displacement field of the membrane of the B&K 4134 microphone is determined for the frequencies 40 kHz and 70 kHz. The results compare favorably with the measured values published in the literature. The paper also proposes a new design, completly micromachined, (the chip including also the back-volume) of the B&Kl/4-inch MEMS measurement microphone. Despite the fact that this new design has a backvolume only 0.8 [mm] deep as compared with 6.4[mm] of the B&K microphone its mechanical response fits very well the measured values published in the literature.

ACKNOWLEDGMENT

This work has been supported by the National Institute On Deafness And Other Communication Disorders Grant No. R01DC005762-05 for a NIH Bioengineering Research Partnership and by Grant Number R01DC009429 to R.N.M. (e-mail: miles@Binghamton.edu). The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institute On Deafness And Other Communication Disorders or the National Institute of Health.

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